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Hybrid Synchronization among Identical Hyperchaotic Systems using Adaptive Control

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Abstract—*In this research work, we describe a systematic methodology for investigating the hybrid synchronization (HS) scheme in two identical hyperchaotic systems. An adaptive control technique (ACT)is proposed which is primarily based on Lyapunov stability theory (LST). The proposed technique concludes the global stability asymptotically and provides parameters identification simultaneously using HS scheme. Further, co-existence of complete and anti-synchronization has been observed in HS scheme. Finally, numerical simulations are implemented to show the feasibility and effectiveness of the considered strategy by using MATLAB. Prominently, the analytic and experimental outcomes agree excellently.*

Keywords: *Adaptive control, hyperchaotic system, hybrid synchronization, Lyapunov stability theory, MATLAB.*

1. Introduction

Chaos is characterized as a highly complex nonlinear phenomenon which exhibits some specific properties such as extremely dependence on the initial conditions, strange attractors, wide Fourier transform spectra, fractal properties in phase space of the motion. A small transition in the system parameters and in the initial conditions result to an extensive variation in the persisting behavior of the system which is the main feature of chaotic and hyperchaotic systems.

Nowadays, chaos synchronization and control of nonlinear chaotic systems have engaged increasing attention in several fields such as biomedical engineering [1], robotics [2], finance models [3], neural networks [4], weather models [5], ecological models [6], oscillations [7], chemical reactions [8], circuits [9], jerk systems [10], encryption [11], etc. Subsequently, chaos synchronization and control have become the prominent branch of applied mathematics intriguing a considerable research interest among several researchers and scientists from varied fields.

Historically, the formulation of chaos theory has been started from Poincare's[12] seminal work in the early 20th century while studying three body problem containing sun, moon and earth in order to examine stability of the solar system. Lorenz [13] reported the first chaotic system while analyzing atmospheric convection model for weather prediction in the year 1963 and also originated the term 'buttery effect' showing the sensitive dependence to the initial conditions. Due to the inadmissible extreme complexity of chaotic systems, the chaos control and synchronization in chaotic systems turn into the primary concern for the scientists and researchers in recent decades.

Pecora and Carroll [14] in the year 1990 firstly investigated the chaos synchronization of nonlinear dynamical systems using a master-slave configuration. Later on, Ott et al.[15] introduced a procedure known as OGY method for controlling chaos in nonlinear dynamical systems in the year 1990.

In chaos synchronization, the output of the slave system traces the out-turn of the master system. The convergence of synchronization error to zero asymptotically with time shows the occurrence of synchronization of two chaotic systems. Till now, numerous types of synchronization techniques for integer order chaotic systems have been developed among nonlinear dynamical systems, for example, complete synchronization [16], hybrid synchronization [17], anti-synchronization [18], hybrid projective synchronization [19], phase synchronization[20], function projective synchronization [21], lag synchronization [22], projective synchronization [23], modified projective synchronization [24] etc.

Up to now, a variety of schemes have been introduced in control theory to stabilize the chaos generated among chaotic systems. Some of them are illustrated as active control [25], feedback control [26], adaptive control [27], backstepping design [28], sliding mode control [29], impulsive control [30] etc.

In the current literature, a hyperchaotic system is simply defined as a chaotic system consisting of at least two positive Lyapunov exponents. An integer order dynamical system is said to be chaotic if the dimension of the state equations must be at least three and having at least one positive Lyapunov exponent. Rossler [31] discovered the first typical hyperchaotic system in the year 1979. In the past four decades, many classical hyperchaotic systems have been developed, for example, Lorenz system, Nikolov system, Chen system, Liusystem, and so on. The exploration of hyperchaos is still in its beginning stage. As a consequence, hyperchaos has acquired a considerable interest from varied scientific and engineering communities.

Chaos synchronization for chaotic systems using ACT was firstly 'discovered in 1989 by Hubler [32]. In [33], the synchronization of chaotic systems, for example, Chua's circuit and Rossler-like system are investigated separately by using ACT and also it is displayed through numerical outcomes that it is applicable in secure communications. In [34], synchronization and controlling of a modified Chua's circuit system by applying ACT is discussed. Moreover, projective synchronization and its application in secure communication are explored in [35]. Furthermore, in [36], adaptive backstepping technique for synchronizing nonlinear chaotic systems is studied. Moreover, in [37], complex projective synchronization for complex chaotic systems has been discussed. In [38], ACT is discussed in detail to synchronize hybridly the generalized Lotka-Voltera three species biological system. In [39, 40, 41], a detailed study of numerous control approaches has been done for newly designed hyperchaotic systems.

Considering the above stated discussions, the current paper focuses on to investigate a hybrid synchronization (HS) scheme in two identical hyperchaotic systems via ACT. ACT is very significant to identify the parameters among master and slave systems. Hence, by using this approach, a less information is required for synchronizing the master and slave systems. Furthermore, an adaptive control law and an estimated parameter update laws have been designed in view of LST.

The paper is arranged as follows: Section 2 contains the preliminaries along with few notations and basic terminology utilized within this paper. In Section 3, methodology of ACT has been elaborated. Further, this section consists of the basic structured characteristics of the given system. Section4 investigates ACT to stabilize globally and asymptotically the considered hyperchaotic system together with a parameter identification update laws. Section 5 contains the numerical simulations to demonstrate the efficiency and feasibility of the discussed HS scheme. Also, a comparison analysis has been discussed. Finally, some concluding remarks are provided in Section 6.

2. Preliminaries

This section provides some notations and terminology and mentions few basic results to be used in the upcoming sections of this paper. Considering the master system and the corresponding slave system as:

$$\dot{p}_1 = f(p_1) \qquad (1)$$

$$\dot{q}_1 = g(q_1) + v \qquad (2)$$

where $p_1 = (p_{11}, p_{12}, \dots, p_{1n})^T$, $q_1 = (q_{11}, q_{12}, \dots, q_{1n})^T$ are the state vectors of (1) and (2) respectively, $f, g : R^n \to R^n$ are two nonlinear continuous vectorfunctions and $v = (v_{11}, v_{12}, \dots, v_{1n}) \in R^n$ is the proper controller to be determined.

Definition 2.1: The master system (1) and the slave system (2) are said to be in hybrid synchronization (HS) if

$$\lim_{t\to\infty}\|e\| = lim_{t\to\infty}||eq_1 - \alpha p_1\| = 0 \qquad (3)$$

for some $\alpha = diag(1, -1, \dots, (-1)^{n+1})$ and $\|.\|$ represents vector norm.

Remark 2.2: The co-existence of complete and anti-synchronization has been observed in HS scheme.

3. System Description

Proposed by Dong et.al.[42], the discussed hyperchaotic system is expressed as:

$$\dot{p}_{11} = m_1 p_{11} - m_2 q_{11} r_{11}$$

$$\dot{q}_{11} = -m_3 q_{11} + p_{11} r_{11}$$

$$\dot{r}_{11} = m_6 p_{11} - m_4 r_{11} + p_{11} q_{11}$$

$$\dot{s}_{11} = m_5 s_{11} + p_{11} q_{11}, \qquad (4)$$

where$(p_{11}, q_{11}, r_{11}, s_{11})^T \in R^4$ is the state vector and m_1, m_2, m_3, m_4, m_5 and m_6 are positive parameters. If $m_1 = 4.55, m_2 = 1{:}532, m_3 = 10.1, m_4 = 5.5, m_5 = 0.04$ and $m_6 = 3.5$,then the given system (4) depicts hyperchaos. Moreover,

Figure 1(a-f) exhibit the phase plots of (4). Further, the thorough analytic study and numerical results for the system (4) may be found in [41].

In the next section, we shall explain the HS scheme to control and synchronize hyperchaos found in (4) using ACT.

4. Illustrative example

Here, we investigate the HS scheme to obtain the laws in order to identify parameters along with adaptive control in such a way that each state variables p_{11}; q_{11}; r_{11} and s_{11} tend to equilibrium points as t approaches to infinity.

Conveniently, the system (4) is treated as the master system and the corresponding slave system may be described as:

$$\dot{p}_{21} = m_1 p_{21} - m_2 q_{21} r_{21} + v_{11}$$

$$\dot{q}_{21} = -m_3 q_{21} + p_{21} r_{21} + v_{12}$$

$$\dot{r}_{21} = m_6 p_{21} - m_4 r_{21} + p_{21} q_{21} + v_{13}$$

$$\dot{s}_{21} = m_5 s_{21} + p_{21} q_{21} + v_{14}, \qquad (5)$$

where v_{11}, v_{12}, v_{13} and v_{14} are controllers to be constructed in such a way thatHS among two identical hyperchaotic systems will be attained.

For that, the state errors are defined as

$e_{11} = p_{21} - p_{11}$

$e_{12} = q_{21} + q_{11}$

$e_{13} = r_{21} - r_{11}$

$e_{14} = s_{21} + s_{11}$ (6)

The prime objective of this section is to design the controllers $v_{1i}, (i = 1, 2, 3, 4)$and parameter update laws accordingly so that the state errors defined in (6) satisfy $lim_{t\to\infty} e_{1i}(t) = 0$ for $(i = 1, 2, 3, 4)$.

The consequent error dynamics is written as:

$e_{11} = m_1 e_{11} - m_2(q_{21} r_{21} - q_{11} r_{11}) + v_{11}$

$e_{12} = - m_3 e_{12} + p_{21} r_{21} + p_{11} r_{11} + v_{12}$

$e_{13} = m_6 e_{11} - m_4 e_{13} + p_{21} q_{21} - p_{11} q_{11} + v_{13}$

$e_{14} = m_5 e_{14} + p_{21} q_{21} + p_{11} q_{11} + v_{14}$ (7)

Now, we construct adaptive controllers as:

$v_{11} = - \widehat{m}_1 e_{11} + \widehat{m}_2(q_{21} r_{21} - q_{11} r_{11}) - L_1 e_{11}$

$v_{12} = \widehat{m}_3 e_{12} - p_{21} r_{21} - p_{11} r_{11} - L_2 e_{12}$

$v_{13} = -\widehat{m}_6 e_{11} + \widehat{m}_4 e_{13} - (p_{21} q_{21} - p_{11} q_{11}) - L_3 e_{13}$

$v_{14} = -\widehat{m}_5 e_{14} - p_{21} q_{21} - p_{11} q_{11} - L_4 e_{14}$ (8)

where$\widehat{m}_1, \widehat{m}_2, \widehat{m}_3, \widehat{m}_4, \widehat{m}_5, \widehat{m}_6$ are estimated values of uncertain parameters $m_1, m_2, m_3, m_4, m_5, m_6$ respectively and L_1, L_2, L_3 and L_4 are positive gainconstantsBy putting the values of adaptive controllers (8) in the error dynamics (7), we obtain

$\dot{e}_{11} = (m_1 - \widehat{m}_1)e_{11} - (m_2 - \widehat{m}_2)(q_{21} r_{21} - q_{11} r_{11}) - L_1 e_{11}$

$\dot{e}_{12} = - (m_3 - \widehat{m}_3)e_{12} - L_2 e_{12}$

$\dot{e}_{13} = (m_6 - \widehat{m}_6)e_{11} - (m_4 - \widehat{m}_4)e_{13} - L_3 e_{13}$

$\dot{e}_{14} = (m_5 - \widehat{m}_5)e_{14} - L_4 e_{14}$ (9)

We describe parameter estimation error as:

$\widetilde{m}_1 = m_1 - \widehat{m}_1, \widetilde{m}_2 = m_2 - \widehat{m}_2$

$\widetilde{m}_3 = m_3 - \widehat{m}_3, \widetilde{m}_4 = m_4 - \widehat{m}_4$

$\widetilde{m}_5 = m_5 - \widehat{m}_5, \widetilde{m}_6 = m_6 - \widehat{m}_6$ (10)

By considering (10), the error dynamics (9) is expressed as:

$\dot{e}_{11} = \widetilde{m}_1 e_{11} - \widetilde{m}_2(q_{21} r_{21} - q_{11} r_{11}) - L_1 e_{11}$

$\dot{e}_{12} = - \widetilde{m}_3 e_{12} - L_2 e_{12}$

$\dot{e}_{13} = \widetilde{m}_6 e_{11} - \widetilde{m}_4 e_{13} - L\_3 e_{13}$

$\dot{e}_{14} = \widetilde{m}_5 e_{14} - L_4 e_{14}$ (11)

On differentiation, the parameter estimation error (10) may be written as

$\dot{\widetilde{m}}_1 = -\dot{\widehat{m}}_1, \dot{\widetilde{m}}_2 = -\dot{\widehat{m}}_2,$

$\dot{\widetilde{m}}_3 = -\dot{\widehat{m}}_3, \dot{\widetilde{m}}_4 = -\dot{\widehat{m}}_4,$

$\dot{\widetilde{m}}_5 = -\dot{\widehat{m}}_5, \dot{\widetilde{m}}_6 = -\dot{\widehat{m}}_6$ (12)

Constructing Lyapunov function as

$V = \frac{1}{2}[e_{11}^2 + e_{12}^2 + e_{13}^2 + e_{14}^2 + \widetilde{m}_1^2 + \widetilde{m}_2^2 + \widetilde{m}_3^2 + \widetilde{m}_4^2 + \widetilde{m}_5^2 + \widetilde{m}_6^2]$ (13)

which shows that V is positive definite .

Derivative of V is described as

$\dot{V} = e_{11}\dot{e}_{11} + e_{12}\dot{e}_{12} + e_{13}\dot{e}_{13} + e_{14}\dot{e}_{14} - \widetilde{m}_1 \dot{\widehat{m}}_1$
$-\widetilde{m}_2 \dot{\widehat{m}}_2 - \widetilde{m}_3 \dot{\widehat{m}}_3 - \widetilde{m}_4 \dot{\widehat{m}}_4 - \widetilde{m}_5 \dot{\widehat{m}}_5 - \widetilde{m}_6 \dot{\widehat{m}}_6$ (14)

Taking (14) into consideration, we design the parameter identification laws as

:

$\dot{\widehat{m}}_1 = (p_{21} - p_{11})e_{11} + L_5(m_1 - \widehat{m}_1)$

$\dot{\widehat{m}}_2 = - (q_{21} r_{21} - q_{11} r_{11})e_{11} + L_6(m_2 - \widehat{m}_2)$

$\dot{\widehat{m}}_3 = - (q_{21} + q_{11})e_{12} + L_7(m_3 - \widehat{m}_3)$

$\dot{\widehat{m}}_4 = - (r_{21} + r_{11})e_{13} + L_8(m_4 - \widehat{m}_4)$

$\dot{\widehat{m}}_5 = (s_{21} + s_{11})e_{14} + L_9(m_5 - \widehat{m}_5)$

$\dot{\widehat{m}}_6 = (p_{21} - p_{11})e_{13} + L_{10}(m_6 - \widehat{m}_6),$ (15)

where L_5, L_6, L_7, L_8, L_9 and L_{10} are positive gain constants.

Theorem 1: The hyperchaotic systems (4)-(5) are hybrid synchronized globally and asymptotically for each initial states $(p_{11}(0), q_{11}(0), r_{11}(0), s_{11}(0)) \in R^4$ by the adaptive controller (8) and the parameter update law (15).

Proof. The Lyapunov function V as described in (13) is a positive definite function. On simplifying equations (14) and (15), we have $\dot{V} = -L_1 e_{11}^2 - L_2 e_{12}^2 - L_3 e_{13}^2 - L_4 e_{14}^2 - L_5 \widetilde{m}_1^2 - L_6 \widetilde{m}_2^2 - L_7 \widetilde{m}_3^2 - L_8 \widetilde{m}_4^2 - 9\widetilde{m}_5^2 - L_{10} \widetilde{m}_6^2 < 0$ which concludes that $\dot{V}$ is negative definite. Therefore, by using LST, we deduce that the HS error $e(t) \to 0$ asymptotically as $t \to \infty$ for each initial conditions $e(0) \in R^4$. This completes the proof.

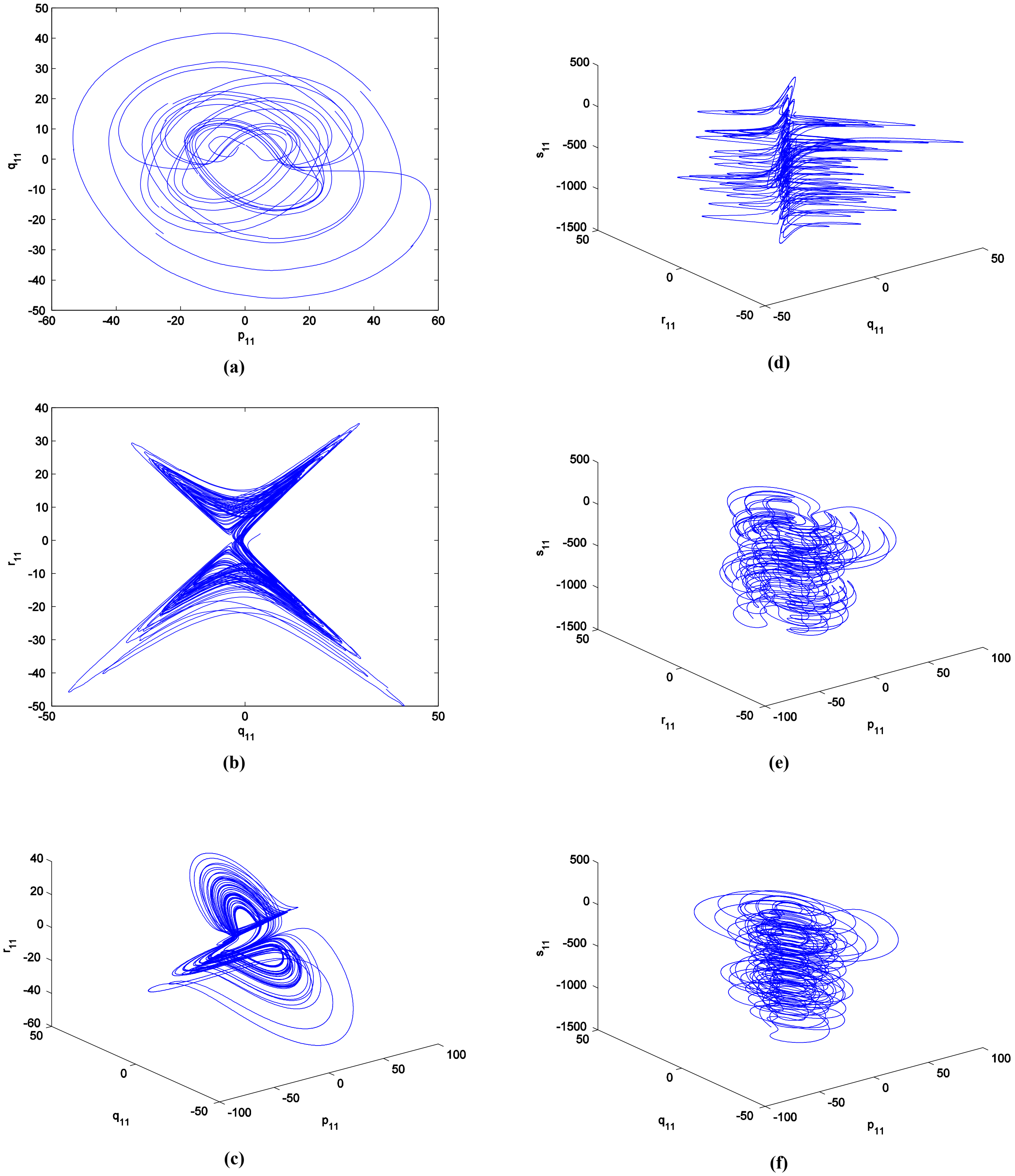
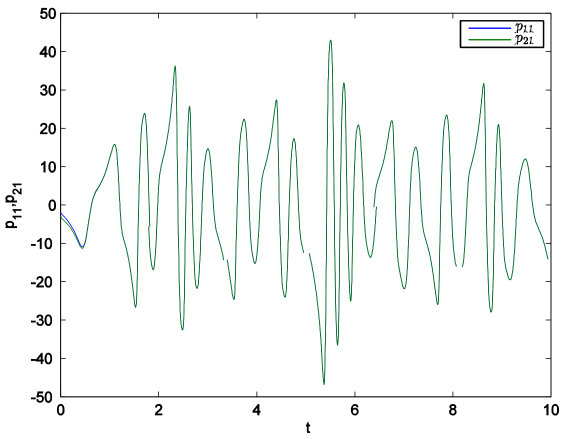


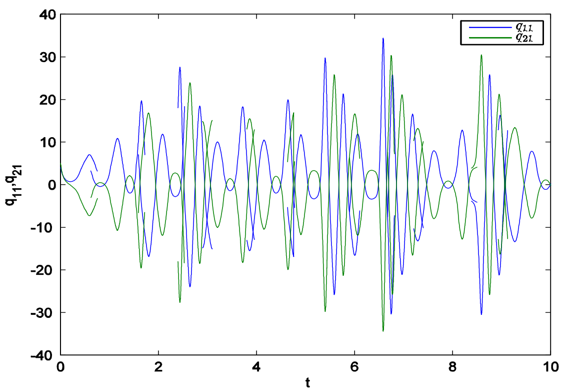
Figure 1: Phase plots of hyperchaotic system in (a) $p_{11} - q_{11}$ plane, (b) $q_{11} - r_{11}$ plane, (c) $p_{11} - q_{11} - r_{11}$ space, (d) $q_{11} - r_{11} - s_{11}$ space, (e) $p_{11} - r_{11} - s_{11}$ space, (f) $p_{11} - q_{11} - s_{11}$ space

5. Numerical Simulations

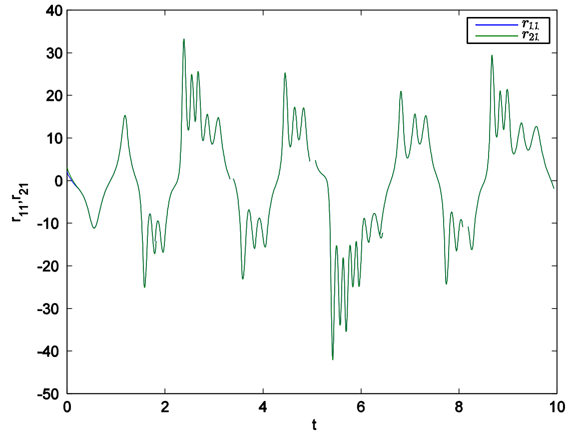
In this section, some simulation results are presented for illustrating the effectiveness and accuracy of the proposed HS scheme using ACT. Here, we utilize the fourth order Runge-Kutta method for solving system of differential equations. The parameters of the considered system are selected as $m_1 = 4.55, m_2 = 1.532, m_3 = 10.1, m_4 = 5.5, m_5 = 0.04$ and $m_6 = 3.5$ to confirm that the system behaves hyper chaotically in the absence of controllers. The initial states of the master and slave systems are $(-2, 4, 2, -3)$ and $(-3, 5, 3, -4)$ respectively. When the scaling matrix α is taken as $\alpha_1 = 1, \alpha_2 = -1, \alpha_3 = 1, \alpha_4 = -1$. The control gains are selected as $L_i = 6$ for $i = 1, 2, \ldots, 10$. The simulation outcomes are displayed in Figure 2(a-d) which exhibit the state trajectories of master and slave systems. Further, in Figure 2(f), it is noticed that the estimated values $(\widehat{m}_1, \widehat{m}_2, \widehat{m}_3, \widehat{m}_4, \widehat{m}_5, \widehat{m}_6)$ of unknown parameters converge to their original values asymptotically in accordance with time. Moreover, synchronization error $(e_{11}, e_{12}, e_{13}, e_{14}) = (1, 9, 1, 7)$ converges to zero as t approaches infinity has been depicted in Figure 2(e). Hence, the investigated HS scheme among master and slave system is verified computationally.



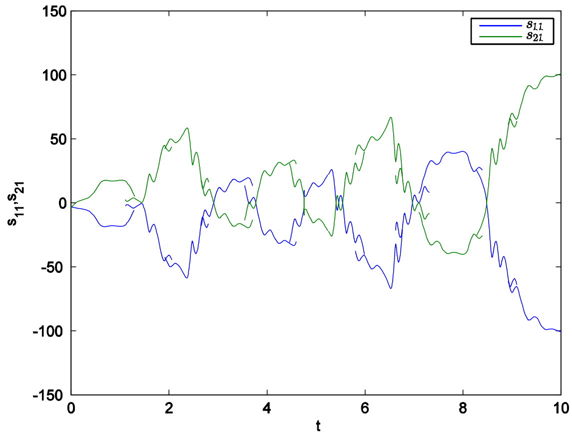
(a)



(b)



(d)



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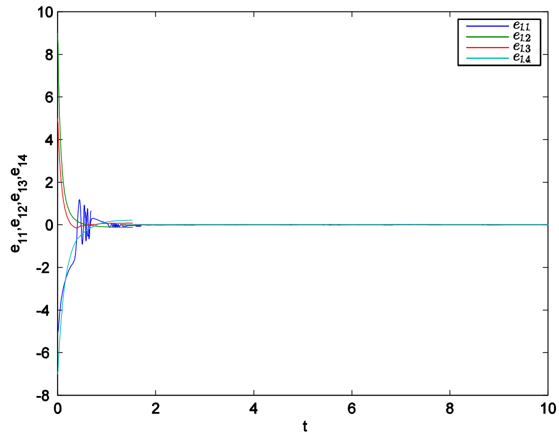
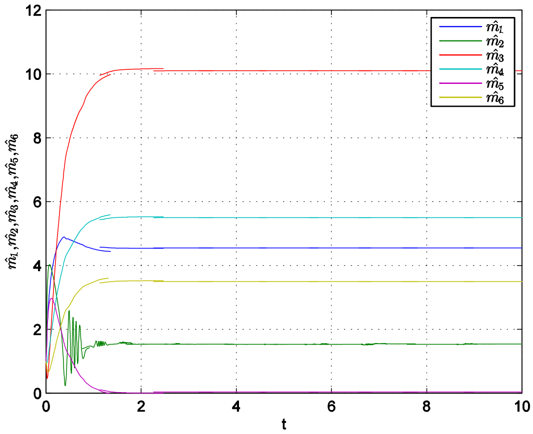


(e)

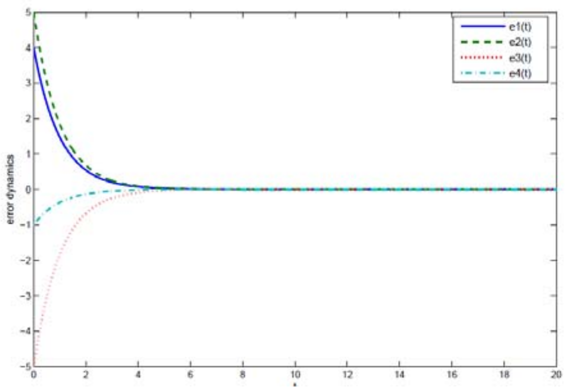


(f)

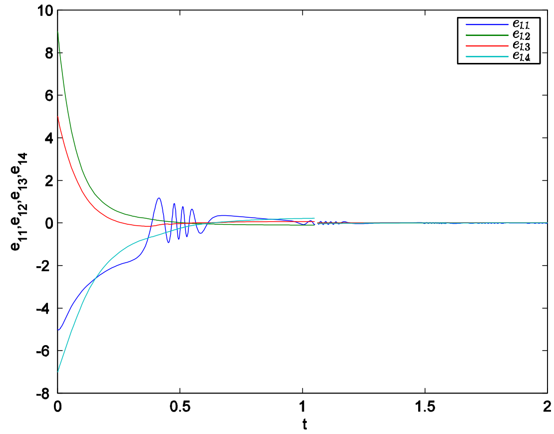
Figure 2: Hybrid synchronization of hyperchaotic system (a) between $p_{11}(t) - p_{21}(t)$, (b) between $q_{11}(t) - q_{21}(t)$, (c) between $r_{11}(t) - r_{21}(t)$, (d) between $s_{11}(t) - s_{21}(t)$, (e) Synchronizationerror, (f) Parameter estimation

5.1 Comparison analysis in the proposed HS scheme and the related earlier published work

Hybrid projective scheme is attained in [40] using active control method when performed on the given hyperchaotic system with identical parameters. It is observed that synchronization error converges to zero at $t = 5.1$(approx) as displayed in Figure 3(a), whereas in this study, the HS scheme is achieved using ACT, in which it is noted that the synchronization error converges to zero at$t = 1.2$(approx) as exhibited in Figure 3(b). This confirms that our proposed HS scheme via ACT is preferred over previously published work.



(a)



(b)

Figure 3: Hybrid synchronization error

6. Conclusion

In this work, the proposed HS scheme among identical hyperchaotic systems via adaptive control technique has been investigated. By designing appropriate adaptive controllers based on LST, the considered HS scheme is attained.

The effectivity and viability of the analytical results have been illustrated in simulations by utilizing MATLAB. Remarkably, the theoretical and the numerical results both are in excellent agreement. Furthermore, the considered HS scheme is very effective as it has numerous applications in image encryption and secure communications. In this work, the time taken by the synchronized error converges to zero is less in comparison with prior related published work. In the end, we understand that the investigated HS scheme via ACT may be generalized by utilizing other designed control techniques.

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